Q1a

1a

For each of the following, find $\frac{dy}{dx}$ in terms of x:

(a) $y = -3x^3 + 5x^2 - 3x + \sqrt{13}$ [2]

(b) $y = 9x^{\frac{1}{3}} - 6x^{-\frac{1}{3}}$ [2]

a) $\frac{dy}{dx} = -3(3)x^{3-1} + 5(2)x^{2-1} - 3(1)x^{1-1} + 0$

$= \boxed{-9x^2 + 10x - 3}$

Q1b

1b

For each of the following, find $\frac{dy}{dx}$ in terms of x:

(a) $y = -3x^3 + 5x^2 - 3x + \sqrt{13}$ [2]

(b) $y = 9x^{\frac{1}{3}} - 6x^{-\frac{1}{3}}$ [2]

b) $\frac{dy}{dx} = 9\left(\frac{1}{3}\right)x^{\frac{1}{3}-1} - 6\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1}$

$= \frac{9}{3}x^{-\frac{2}{3}} + \frac{6}{3}x^{-\frac{4}{3}}$

$= \boxed{3x^{-\frac{2}{3}} + 2x^{-\frac{4}{3}}}$

Q2

2

Given that $y = \frac{1}{\sqrt{x}}\left(1 + \frac{1}{x}\right)$, $x > 0$, find $\frac{dy}{dx}$.

[3]

$y = \frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

$= x^{-\frac{1}{2}} + x^{-3/2}$

It's helpful to rewrite terms as fractional powers of x before differenciating.

$\frac{dy}{dx} = \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + \left(-\frac{3}{2}\right)x^{-\frac{3}{2}-1}$

$= \boxed{-\frac{1}{2}x^{-\frac{3}{2}} - \frac{3}{2}x^{-5/2}}$

Q3a

3a

For each of the following, find $\frac{dy}{dx}$ in terms of x:

(a) $y = (2x - 1)^2(x + 1)$

[3]

(b) $y = \frac{1}{x^5}(x^2 + \sqrt{x} - 1)$

[3]

a) $y = (2x-1)(2x-1)(x+1)$

$= (4x^2 - 4x + 1)(x+1)$

$= 4x^3 - \cancel{4x^2} + x + \cancel{4x^2} - 4x + 1$

$= 4x^3 - 3x + 1$

$\frac{dy}{dx} = 4(3)x^{3-1} - 3(1)x^{1-1} + 0$

$= \boxed{12x^2 - 3}$

Q3b

3b

For each of the following, find $\frac{dy}{dx}$ in terms of x:

(a) $y = (2x-1)^2(x+1)$ [3]

(b) $y = \frac{1}{x^5}(x^2 + \sqrt{x} - 1)$ [3]

b) $y = \frac{x^2}{x^5} + \frac{x^{\frac{1}{2}}}{x^5} - \frac{1}{x^5}$

$= x^{-3} + x^{-9/2} - x^{-5}$

$\frac{dy}{dx} = -3x^{-3-1} + -\frac{9}{2}x^{-\frac{9}{2}-1} - (-5)x^{-5-1}$

$= \boxed{-3x^{-4} - \frac{9}{2}x^{-\frac{11}{2}} + 5x^{-6}}$

Q4a

4a

For each of the following, use the chain rule to find $\frac{dy}{dx}$ in terms of x:

(a) $y = \left(\frac{1}{x} + 2x\right)^4$ [4]

$u = x^{-1} + 2x$

(b) $y = \frac{1}{(x^3-1)^2}$ [4]

a) $u = x^{-1} + 2x$ $\quad y = u^4$ $\quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{du}{dx} = -x^{-2} + 2$ $\quad \frac{dy}{du} = 4u^3$

$\frac{dy}{dx} = 4u^3(2 - x^{-2})$

$\boxed{\frac{dy}{dx} = 4\left(\frac{1}{x} + 2x\right)^3\left(2 - \frac{1}{x^2}\right)}$

Q4b

4b

For each of the following, use the chain rule to find $\frac{dy}{dx}$ in terms of x:

(a) $y = \left(\frac{1}{x} + 2x\right)^4$ [4]

(b) $y = \frac{1}{(x^3-1)^2} = \frac{1}{u^2} = u^{-2}$ [4]

u

b) $u = x^3 - 1$ $\quad y = u^{-2}$

$\frac{du}{dx} = 3x^2$ $\quad \frac{dy}{du} = -2u^{-3}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = -2u^{-3}(3x^2) = -6u^{-3}x^2$

$= -6(x^3-1)^{-3}x^2$

$\boxed{\frac{dy}{dx} = \frac{-6x^2}{(x^3-1)^3}}$

Q5

5

The function f is defined by $f(x) = x^3 - 4x^2 + 6x - 9$. Show that there are no solutions to the equation $f'(x) = 0$. [4]

$f'(x) = 3x^2 - 8x + 6 = 0$

discriminant: $b^2 - 4ac$

$= (-8)^2 - 4(3)(6) = 64 - 72 = -8$

$\boxed{-8 < 0 \quad \therefore \text{ no real solutions.}}$

Q6a

6a

A curve has the equation $y = \frac{3}{8}x^{\frac{4}{3}} - 12x^{\frac{1}{3}}$.

(a) Show that $\frac{dy}{dx} = ax^{-\frac{2}{3}}(x + b)$, where a and b are rational numbers to be found.

[3]

(b) Hence find the coordinates of the point on the curve where the gradient is 0.

[2]

a) $\frac{dy}{dx} = \frac{3}{8}\left(\frac{4}{3}\right)x^{\frac{4}{3}-1} - 12\left(\frac{1}{3}\right)x^{\frac{1}{3}-1}$

$= \frac{1}{2}x^{\frac{1}{3}} - 4x^{-\frac{2}{3}}$ ①

$= x^{-\frac{2}{3}}\left(\frac{1}{2}\frac{x^{\frac{1}{3}}}{x^{-2/3}} - \frac{4\cancel{x^{-2/3}}}{\cancel{x^{-2/3}}}\right)$

$= x^{-\frac{2}{3}}\left(\frac{1}{2}x - 4\right)$

$= \frac{1}{2}x^{-\frac{2}{3}}(x - 8)$

$\boxed{a = \frac{1}{2},\ b = -8}$

...alternatively, working backwards from the eqn in the question:

$\frac{dy}{dx} = ax^{\frac{1}{3}} + bax^{-\frac{2}{3}}$

compare coefficients with ①

$a = \frac{1}{2}$

$ba = -4$

$b = -8$

Q6b

6b

A curve has the equation $y = \frac{3}{8}x^{\frac{4}{3}} - 12x^{\frac{1}{3}}$.

(a) Show that $\frac{dy}{dx} = ax^{-\frac{2}{3}}(x + b)$, where a and b are rational numbers to be found.

$a = \frac{1}{2}$ $b = -8$

[3]

(b) Hence find the coordinates of the point on the curve where the gradient is 0.

[2]

b) $\frac{dy}{dx} = 0 = \frac{1}{2}x^{-\frac{2}{3}}(x - 8)$

$= \frac{1}{2}\,\frac{x - 8}{x^{2/3}}$

$x - 8 = 0$

$\therefore x = 8$

sub $x = 8$

$y = \frac{3}{8}(8)^{\frac{4}{3}} - 12(8)^{\frac{1}{3}} = -18$

POINT: $\boxed{(8, -18)}$

Q7a

7a

Given that $y = (x^2 - 2x - 3)^4$
u

(a) use the chain rule to find $\frac{dy}{dx}$ [3]

(b) find the coordinates of any stationary points and determine their nature [4]

(c) sketch the curve. [3]

a) $u = x^2 - 2x - 3 \qquad y = u^4$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{du}{dx} = 2x - 2 \qquad \frac{dy}{du} = 4u^3$

$\frac{dy}{dx} = 4u^3(2x-2)$

Sub in $u = x^2 - 2x - 3$

$= 4(x^2 - 2x - 3)^3(2x - 2)$

$$\boxed{\frac{dy}{dx} = 8(x^2-2x-3)^3(x-1)}$$

Q7b

7b

Given that $y = (x^2 - 2x - 3)^4$

(a) use the chain rule to find $\frac{dy}{dx}$ [3]

$\frac{dy}{dx} = 8(x^2 - 2x - 3)^3(x-1)$

(b) find the coordinates of any stationary points and determine their nature [4]

(c) sketch the curve. [3]

b) $\frac{dy}{dx} = 0 = 8(x^2 - 2x - 3)^3(x-1)$

$0 = 8(x-3)^3(x+1)^3(x-1)$

$x = 3, -1, 1$

When $x = 3$, $y = (3^2 - 2(3) - 3)^4 = 0$

$x = -1$, $y = ((-1)^2 - 2(-1) - 3)^4 = 0$

$x = 1$, $y = (1^2 - 2(1) - 3)^4 = 256$

SPs: $(-1,0)$, $(3,0)$, $(1,256)$

Determine nature

x	-2	0	2	4
$\frac{dy}{dx}$	-3000	216	-216	3000

$-$ $+$ $-$ $+$

$(-1,0)$ min point

$(1,256)$ max point

$(3,0)$ min point

$$\boxed{\begin{array}{l}\text{min points at } (-1,0) \text{ and } (3,0)\\ \text{max point at } (1, 256)\end{array}}$$

Q7c

7c

Given that $y = (x^2 - 2x - 3)^4$

(a) use the chain rule to find $\frac{dy}{dx}$

[3]

(b) find the coordinates of any stationary points and determine their nature

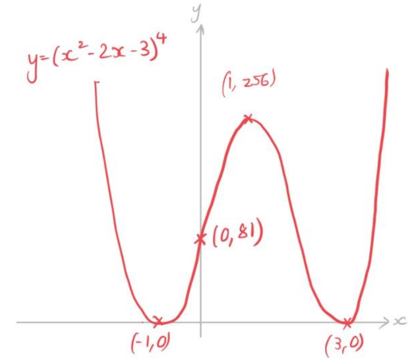
min points: $(-1, 0)$, $(3, 0)$ max point: $(1, 256)$

[4]

(c) sketch the curve.

[3]

c)



y-axis intercept

when $x=0$, $y = (0^2 - 2(0) - 3)^4 = (-3)^4 = 81$

$(0, 81)$

Q8

8

A curve has the equation $y = 4x^3 + bx^2 + 3x - 17$, where b is a constant. Given that there is only one point on the curve where the gradient is zero, determine the possible values of b.

[4]

$\frac{dy}{dx} = 4(3)x^2 + b2x + 3$

$= 12x^2 + 2bx + 3 = 0$

discriminant $= 0$ since quadratic has only one solution

$b^2 - 4ac = 0$

$(2b)^2 - 4(12)(3) = 0$

$b^2 = 36$

$b = \sqrt{36} = \boxed{\pm 6}$

Q9

9

A curve is described by the equation $4y^2 - 3x^5 = 0$, $y > 0$.

By rearranging the equation to make y the subject, find $\frac{dy}{dx}$.

[2]

$y^2 = \frac{3x^5}{4}$

$y = \sqrt{\frac{3x^5}{4}} = \frac{\sqrt{3}}{2}x^{5/2}$ ← rewrite the power in a form that is easy to differenciate

$\frac{dy}{dx} = \frac{\sqrt{3}}{2}\left(\frac{5}{2}\right)x^{\frac{5}{2}-1} = \boxed{\frac{5\sqrt{3}}{4}x^{\frac{3}{2}}}$

Q10

10

The curve with equation $y = ax^2 + bx + c$ has a gradient of 8 at the point $(-2, 0)$, and a gradient of -10 at the point $(1, -3)$. Find the values of a, b and c.

[5]

$\frac{dy}{dx} = 2ax + b$

at $(-2,0)$ $\quad \frac{dy}{dx} = 2a(-2) + b = \boxed{8 = -4a + b}$ ①

at $(1,-3)$ $\quad \frac{dy}{dx} = 2a(1) + b = \boxed{-10 = 2a + b}$ ②

Solve simultaneous eqns to find a and b.

① − ②

$-6a = 18 \quad \therefore a = -3$

Sub $a = -3$ into ① (or ②!)

$-4(-3) + b = 8 \quad \therefore b = -4$

Sub $a = -3$, $b = -4$ and one of the points given into curve.

At $(-2,0)$ $\quad (0) = (-3)(-2)^2 + (-4)(-2) + c$

$= -12 + 8 + c$

$\therefore c = 4$

$\boxed{a = -3 \quad b = -4 \quad c = 4}$